Benha University Faculty of Engineering- Shoubra Eng. Mathematics & Physics Department Preparatory Year		Final Term Exam Date: May 28 , 2018 Course: Mathematics 1 – B Duration: **3** hours

The Exam consists of one page   Answer **All** Questions   No. of questions: 4   Total Mark: 100

Question 1

(a)Find **y`** : (i) $y = \cosh x^2 . \tanh x + \cosh^{-1} x$ (ii) $y = \sin^{-1} x . \tanh^{-2} x - \tan^{-1} x$ 4

(b)Prove that : (i) $\operatorname{sech}^{-1} x = \ln \frac{1+\sqrt{1-x^2}}{x}$. 3

(ii) If $I_n = \int \tan^n x \, dx$. Then $I_n = \frac{1}{n-1} (\tan x)^{n-1} - I_{n-2}$. 3

(c)Find the area of the region inside the parabola : $x^2 - 4x + y = 0$ and above x–axis. 3

(d)Find the arc length of the cycloid : $y = 1 - \cos\theta$, $x = \theta - \sin\theta$, θ in $[0, 2\pi]$. 3

(e)Find the volume of the solid generated by rotating the region between the curves : $y + x = 3$, $y = 6 - 3x - x^2$ about x–axis. 3

(f)If the region bounded by the curve $x = y^2$, y-axis, y in [2, 3] is rotated about y–axis. Find the total surface area of the generated solid. 4

Question 2

27

Find the following integrals :

(a) $\int \left(\frac{2}{1+x^2} - \frac{3}{\sqrt{1+x^2}}\right) dx$ (b) $\int \left(\frac{2^x}{1+2^x} + \frac{1}{1-e^x}\right) dx$ (c) $\int x \cosh x \, dx$

(d) $\int \tan^{-1} 3x \, dx$ (e) $\int \frac{x}{x^2-6x+10} dx$ (f) $\int \frac{1}{5+4\cos x} dx$

(g) $\int \frac{\cos x}{\sqrt{5+2\sin x}} dx$ (h) $\int (\cos 2t . \sin 3t)^2 \, dt$ (i) $\int_0^{\pi} (\sin y)^9 \, dy$

Question 3

(a)If the equation $ax^2 + 2h\,xy + by^2 = 0$ represents pair of lines such that the slope of one is twice the slope of the other. Prove that $8h^2 = 9ab$. 6

(b)If (4, 1) , (1, –2) are limiting points of coaxial system of circles. Find the equation of orthogonal coaxial system. 7

(c)Find the equation of the tangent to the curve : $9x^2 - 2y^2 + 18x + 12y - 27 = 0$ at the point (1, 6) and sketch the curve. 12

Question 4

(a)Find the equation of the tangent from the point (3, 0) to the curve : $3x^2 + y^2 = 12$. 6

(b)If (1, 2) is one end of a chord of the curve : $y^2 = 4x$ and the equation of diameter is $y = 3$. Find the other end of the chord and the equation of tangent at this point. 7

(c)Find the point of intersection of the line $\frac{x-9}{6} = \frac{y-7}{2} = \frac{z-6}{-3}$ with xz-plane and the shortest distance from the line $\frac{x-4}{3} = \frac{y-4}{2} = \frac{z-1}{-2}$. 12

Good Luck   *Dr. Khaled Elnaggar*   *Dr. Mohamed Eid*

Model Answer

Answer of Question 1

(a)(i) $y` = \cosh x^2 . (\operatorname{sech} x)^2 + \sinh x^2 . 2x . \tanh x + \frac{1}{\sqrt{x^2-1}}$

(ii) $y` = \sin^{-1} x . (-2)(\tanh x)^{-3} . (\operatorname{sech} x)^2 + \frac{1}{\sqrt{1-x^2}} . (\tanh x)^{-2} - \frac{1}{1+x^2}$

-- ***(4 Marks)***

(b)(i), (ii) Proof

-- ***(6 Marks)***

$$\text{(c) } A = \int_0^4 (4x - x^2)\, dx = \frac{32}{3}$$

-- ***(3 Marks)***

(d)Since $\dot{x} = 1 - \cos\theta$, $\dot{y} = \sin\theta$. Then

$$L = \int_0^{2\pi} \sqrt{(\dot{x})^2 + (\dot{y})^2}\, d\theta = \int_0^{2\pi} \sqrt{(1-\cos\theta)^2 + (\sin\theta)^2}\, d\theta = 8$$

-- ***(3 Marks)***

(e)Solving the two equations, we get $x = -3$ and $x = 1$. Then

$$V_x = \pi \int_{-3}^{1} [(y_1)^2 - (y_2)^2]\, dx = \pi \int_{-3}^{1} [(6 - 3x - x^2)^2 - (3-x)^2]\, dx = \frac{1792}{15}\pi = 119.47\,\pi$$

-- ***(3 Marks)***

(f) $S = S_y$ + area of circle with radius 4 + area of circle with radius 9.

$$S_y = 2\pi \int_2^3 x . \sqrt{1 + (x`)^2}\, dy = 2\pi \int_2^3 y^2 . \sqrt{1 + (2y)^2}\, dy = 66.24\,\pi$$

Then $S = 66.24\,\pi + 16\,\pi + 81\,\pi = 163.24\,\pi$

-- ***(4 Marks)***

Answer of Question 2

(a) $I = 2\tan^{-1}x - 3\sinh^{-1}x + c$

(b) $I = \int \frac{2^x}{1+2^x}dx - \int \frac{1-e^x+e^x}{1-e^x}dx = \frac{1}{\ln 2}\ln(1+2^x) - x + \ln(1-e^x) + c$

(c) By parts : $I = x\sinh x - \int \sinh x \; dx = x\sinh x - \cosh x + c$

(d) By parts : $I = x\;\tan^{-1}3x - \int \frac{3x}{1+9x^2}dx = x\;\tan^{-1}3x - \frac{1}{6}\ln(1+9x^2) + c$

(e) $I = \frac{1}{2}\int \frac{2x-6+6}{x^2-6x+10}dx = \frac{1}{2}\int [\frac{2x-6}{x^2-6x+10} + \frac{6}{(x-3)^2+1}]\,dx$

$$= \frac{1}{2}[\ln(x^2-6x+10) + 6\tan^{-1}(x-3)] + c$$

(f)Put $u = \tan\frac{1}{2}x$. Then $I = \int \frac{2}{9+u^2}du = \frac{2}{3}\tan^{-1}\frac{1}{3}u = \frac{2}{3}\tan^{-1}(\frac{1}{3}\tan\frac{1}{2}x) + c$

(g) $I = \frac{1}{2}\int 2\cos x\,.\,(5+2\sin x)^{-\frac{1}{2}}\;dx = \sqrt{5+2\sin x}\; + c$

(h) $\int(\cos 2t\,.\,\sin 3t)^2\,dt = \frac{1}{4}\int(1+\cos 4t)(1-\cos 6t)\,dt$

$$= \frac{1}{4}\int(1+\cos 4t-\cos 6t-\cos 4t\,.\,\cos 6t)\,dt$$

$$= \frac{1}{4}\int\left(1+\cos 4t-\cos 6t-\frac{1}{2}(\cos 2t+\cos 10t)\right)dt$$

$$= \frac{1}{4}(t+\frac{1}{4}\sin 4t-\frac{1}{6}\sin 6t-\frac{1}{4}\sin 2t-\frac{1}{20}\sin 10t) + c$$

(i) $I_9 = 2\frac{8.6.4.2}{9.7.5.3.1} = \frac{256}{315} = 0.81$

--- ***(27 Marks)***

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Answer of Question 3 **[25]**

a) If $a\,x^2 + 2h\,xy + b\,y^2 = 0$ represent pair of straight lines such that the slope of one line is twice the slope of the other.Prove $8\,h^2 = 9\,a\,b$. **[6]**

Answer

The above pair can be factorized into

$(y - m_1 x)(y - m_2 x) = 0 \Rightarrow y^2 - (m_1 + m_2)\,x\,y + m_1 m_2 x^2 = 0.$

Compare the coefficients such that $m_1 m_2 = \frac{a}{b}$ & $m_1 + m_2 = -\frac{2h}{b}$.

Since $m_1 = 2\,m_2 \Rightarrow (2\,m_2)(m_2) = \frac{a}{b}$ & $2\,m_2 + m_2 = -\frac{2h}{b}$.

$\Rightarrow 2\,m_2{}^2 = \frac{a}{b}$ & $3\,m_2 = -\frac{2h}{b} \Rightarrow 2\,(-\frac{2h}{3b})^2 = \frac{a}{b} \Rightarrow 8\,h^2 = 9\,a\,b.$

b) (4, 1) and (1, -2) are 2 limiting points of coaxial system of circles, find the equation of orthogonal coaxial system. **[7]**

Answer

Since (4, 1) and (1, -2) are the 2 points of intersection of the orthogonal coaxial system, therfore (4, 1) and (1, -2) are 2 end of diameter of a circle belong to the orthogonal system $\Rightarrow$ S: $(x - 4)(x - 1) + (y - 1)(y + 2) = 0$ and the radical axis of the orthogonal system is expressed by L : $\frac{y-1}{x-4} = \frac{-2-1}{1-4} = 1 \Rightarrow L : y = x - 3$.

Therefore the equation of orthogonal system is $S + k\,L = 0$

c) Find equation of the tangent to the curve $9\,x^2 - 2\,y^2 + 18\,x + 12\,y - 27 = 0$ at (1, 6) and then sketch the curve. **[12]**

Answer

By completing square, we get $9\,(x + 1)^2 - 2\,(y - 3)^2 = 18 \Rightarrow \frac{(x+1)^2}{2} - \frac{(y-3)^2}{9} = 1$

Put $X = x + 1$ & $Y = y - 3 \Rightarrow \frac{X^2}{2} - \frac{Y^2}{9} = 1$ and the point of contact is (2, 3).

Therefore the equation of tangent is $\frac{2X}{2} - \frac{3Y}{9} = 1 \Rightarrow 3X - Y = 3 \Rightarrow 3\,x - y = -3$.

$a^2 = 2$, $b^2 = 9$ and center of the hyperbola is (-1, 3) and $b^2 = (e^2 - 1)\,a^2 \Rightarrow e^2 = \frac{11}{2}$ $e = \sqrt{\frac{11}{2}}$.

Thus coordinates of focii $(-1 \pm \sqrt{11}, 3)$ and that of vertices $(-1 \pm \sqrt{2}, 3)$, finally the coordinates of directices $x = -1 \pm \frac{2}{\sqrt{11}}$.

Answer of Question 4 **[25]**

a) Find equation of the tangent from (3, 0) to the curve $3x^2 + y^2 = 12$. **[6]**

Answer

(3, 0) is outside the ellipse $\frac{x^2}{4} + \frac{y^2}{12} = 1$, therefore we have to get the equation of the chord of contact such that $x = \frac{4}{3}$ is the chord of contact and then substitute in the equation of ellipse so that we get the 2 points of contact $(\frac{4}{3}, \pm\frac{\sqrt{20}}{3})$.

Hence the equation of the 2 tangents are $\frac{x}{3} \pm \frac{\sqrt{20}\,y}{12\sqrt{3}} = 1$.

b) If (1, 2) is one end of a chord to the curve $y^2 = 4x$ and the equation of diameter is y = 3.Find the other end of the chord and the equation of tangent at that point. **[7]**

Answer

The equation of the diameter is y = 3, therefore the y coordinate of the mid point of the chord = 3 and hence y coordinate of the other end = 4.

But the end points of the chord satisfy the equation $y^2 = 4x$, therefore x coordinate of the other end = 4. Thus (4, 4) is the other end.The equation of tangent is 4 y = 2(x + 4).

c) Find the point of intersection of the line $\frac{x-9}{6} = \frac{y-7}{2} = \frac{z-6}{-3}$ with x z plane, and the shortest distance from the line $\frac{x-4}{3} = \frac{y-4}{2} = \frac{z-1}{-2}$. **[12]**

Answer

Let $\frac{x-4}{3} = \frac{y-4}{2} = \frac{z-1}{-2} = t$ and the point P = (x, y, z) satisfy the line L_1 such that the parametric equation is x = 4 + 3t, y = 4 + 2t, z = 1 – 2t.

Similarly let $\frac{x-9}{6} = \frac{y-7}{2} = \frac{z-6}{-3} = s$ and the point Q = (m, n, r) satisfy the line L_2 such that the parametric equation is m = 9 + 6s, n = 7 + 2s, r = 6 – 3s, hence the line vector $\overrightarrow{PQ}$ joining the 2 points is expressed by $\overrightarrow{PQ} = \overrightarrow{Q} - \overrightarrow{P}$ = (6s – 3t + 5, 2s – 2t + 3, -3s + 2t + 5) such that $\overrightarrow{PQ} \perp L_1$ and $\overrightarrow{PQ} \perp L_2$, i.e. the points P and Q will be nearest to each other, thus 3(6s – 3t + 5) + 2(2s – 2t + 3) – 2(-3s + 2t + 5) = 0 and 6(6s – 3t + 5) + 2(2s – 2t + 3) – 3(-3s + 2t + 5) = 0.

Solve the two equations so that s = -1, t = -1 and therefore P = (1, 2, 3), Q = (3, 5, 9), thus the shortest distance is $\overline{PQ} = \sqrt{2^2 + 3^2 + 6^2}$ = 7

If the line $\frac{x-9}{6} = \frac{y-7}{2} = \frac{z-6}{-3}$ intersect x z plane and the parametric equation of the line is m = 9 + 6s, n = 7 + 2s, r = 6 – 3s therefore 7 + 2s = 0 $\Rightarrow s = -\frac{7}{2}$.

Therefore the point of intersection is (-12, 0, $\frac{33}{2}$)..

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Final Exam and ILOs

Course Title: Mathematics 1-B **Code:** EMP 021

Questions	ILOs		
	Knowledge and Understanding	Intellectual Skills	Professional and Practical Skills
	a.1	b.1	c.1
Q1	√	√	
Q2	√	√	√
Q3	√		√
Q4	√	√	

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